



FOR TORSIONAL STRAIN γ ,

$$\tan \gamma = \frac{dx}{dy},$$

OR, TO A FIRST APPROXIMATION,

$$\gamma = r_m \frac{\phi}{h},$$

WHERE r_m IS THE LIMITING RADIUS.

SUBSTITUTING ω FOR $\frac{\phi}{h}$, AND DEFINING TORSIONAL SHEAR STRENGTH AS,

$$\tau = G \gamma,$$

WHERE G IS THE MODULUS OF RIGIDITY,

WE HAVE

$$\tau_{\max.} = G \omega r_m.$$

THE MOMENT ABOUT THE DISK AXIS IS

$$\begin{aligned} M &= \tau dA r, \\ &= G \omega r^2 dA. \end{aligned}$$

TAKING THE MOMENT OVER THE CROSS SECTION OF THE DISK, WE HAVE

$$M = 2\pi \int_0^{r_m} G \omega r^3 dr.$$

SUBSTITUTING $\tau = G \omega r$,

$$M = 2\pi \int_0^{r_m} \tau r^2 dr,$$

$$M = \frac{2\pi r^3 \tau}{3},$$

AND $\tau_m = \frac{3M}{2\pi r^3}.$

$$M = \frac{\pi G \omega r^4}{2}.$$

SUBSTITUTING $\tau = G \omega r$,

$$M = \frac{\pi \tau r^3}{2},$$

AND $\tau_m = \frac{2M}{\pi r^3}.$